EFFECT OF WAVE PROCESSES ON THE VISCOUS–NONVISCOUS INTERACTION OF A SUB- AND SUPERSONIC JET WITH A SUPER- AND SUBSONIC SECONDARY FLOW IN A CHANNEL AND IN A TUBE

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The study of the interaction of sub- and supersonic jets with secondary super- and subsonic flows in channels and in tubes is of great practical interest, particularly in connection with the processes of turbulent mixing and interaction as a consequence of pressure. The most complete mathematical model of such flows is the one based on the Navier–Stokes equations, enhanced with equations describing turbulent transfer. However, the numerical solution of the Navier–Stokes equations requires the expenditure of considerable amounts of computer time [1] and raises certain methodological difficulties in the area of high Reynolds numbers [2]. The complexity of the situation is compounded in an investigation of turbulent flows. This is a result, on the one hand, of the need to resort to empirical information to close the existing theories of elasticity [3], and on the other hand, by the continued increasing complexity of the system of differential equations describing the averaged turbulent flows and their microstructure [4]. Therefore, in practical calculations we use simplified approaches that are based on the numerical solution of regular finite-difference methods of parabolized Navier–Stokes equations [2, 5]. Since the area of effective utilization of parabolized Navier–Stokes equations is limited primarily to supersonic flows, in order to calculate the injection of subsonic jets into a secondary supersonic flow, boundary-layer equations have found extensive applications. These are equations of the parabolic type and they make possible solutions by standard finite-difference methods for both super- and subsonic flows. Most effective are the boundary-layer equations used to calculate sub- and supersonic jets in a secondary supersonic flow in tubes and in channels [6, 7], the distribution of pressure in which is found by proceeding from the condition of the conservation of mass. Such an approach allows us to find a good approximate solution of the problem for the case in which the wave processes in a nonviscous supersonic flow can be ignored (for example, in the case of low nontheoretical regimes of supersonic jet discharge or in the discharge of a subsonic jet of great intensity, when the pressure at the outlet from the nozzle is close to the pressure in the secondary flow).

In the injection of sub- and supersonic jets into a secondary super- and subsonic flow within a channel or a tube, the greater portion of which is occupied by the supersonic flow, complete solution of the problem requires detailed computation of the interaction between the pressure of the injected jet and the outer flow. In this case, the model using the boundary-layer equations to describe the flow in a subsonic jet or flow and the equations of nonviscous flow (the Euler equations) for a supersonic external flow or for the injected jet proved to be most effective. The compiled portion of this model includes relationships which describe the viscous–nonviscous interaction and the solutions of the differential equations which arise out of asymptotic joining. The methodology for the solution of such problems was developed in [8-10]. In the present study, on the basis of this methodology, we investigate the influence of wave processes in a nonviscous flow on the characteristics of flow in the viscous region on interaction of a sub- and supersonic jet with secondary super- and subsonic flows in a channel and in a tube.

1. We examined the two-dimensional flow in a channel or in a tube on the interaction of a sub- or supersonic injected jet with an external super- or subsonic flow. The gas in the flow and in the jet is assumed to be uniform in composition and calorically and thermally perfect. The deceleration temperatures within these flows may generally be quite different. The diagram for the discharge of a subsonic jet into a secondary supersonic flow in a channel is shown in Fig. 1. The calculation of the flow is accomplished within

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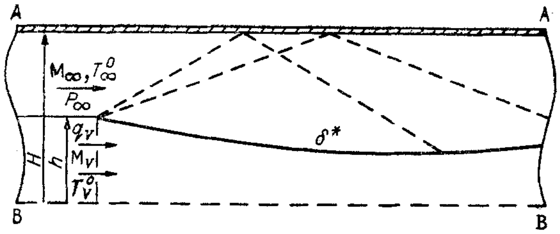

Fig. 1

the framework of the model of viscous—nonviscous interaction, employed earlier in the study of jet flows in problems related to external aerogasdynamics [8-10]. According to this model, the flow is conditionally broken down into a nonviscous flow which streamlines an effective displacement body, and into a viscous flow in the zone in which the jet mixes with the secondary flow, described in approximation of the boundary layer. The characteristics of the nonviscous flow in the inlet section, corresponding to the cross section of the nozzle outlet, are assumed to be known, while the values of the gasdynamic parameters downstream are found by numerical integration of the Euler equations.

In contrast to classical Prandtl boundary layer theory, in which the distribution of pressure along the viscous-flow region is equal to the local pressure of the nonviscous flow, in the calculation of jet and separation flows within the scope of the viscous—nonviscous interaction model it is necessary, even in first approximation, to account for the influence of the viscosity-induced pressure gradient on the characteristics of the nonviscous flow. A consistent value for the pressure gradient is determined through combined calculation of the viscous and nonviscous flows by means of additional equations, i.e., the conditions of viscous—nonviscous interaction [8-10]. These conditions form to make up a system of ordinary differential equations for the pressure in the viscous region and at the boundaries of an effective displacement body. According to [8, 9], in a cylindrical coordinate system for two-dimensional flows, these are written in the form

$$\left[\int_0^\delta \left(1-\frac{a^2}{u^2}\right) y^j\,dy - \int_{\delta^*}^\delta \left(1-\frac{a_e^2}{u_e^2}\right) y^j\,dy\right]\frac{1}{\gamma P_e}\frac{dP_e}{dx} - \delta^{*j}\frac{d\delta^*}{dx} = \tag{1.1}$$

$$= \frac{\gamma-1}{\gamma P_e}\int_0^\delta \frac{1}{u^2}\left(u\frac{\partial Q}{\partial y} - h\frac{\partial \tau}{\partial y}\right) dy;$$

$$q\frac{dP_e}{dx} - \rho_e u_e^2 \frac{dq}{dx} = \left.\frac{\partial P}{\partial y}\right|_{y=\delta^*}(1+q^2); \tag{1.2}$$

$$d\delta^*/dx = q, \tag{1.3}$$

where

$$\frac{\partial Q}{\partial y} = \frac{\partial}{\partial y}\left(y^j \frac{\rho \nu_t}{\mathrm{Pr}_t}\frac{\partial h}{\partial y}\right) + y^j \rho \nu_t \left(\frac{\partial u}{\partial y}\right)^2; \quad \tau = y^j \rho \nu_t \frac{\partial u}{\partial y};$$

ν_t is the turbulent viscosity; $\delta(x)$ and $\delta^*(x)$ are the conditional boundary of the viscous region and the boundary of the effective displacement body; $j = 0$ corresponds to plane flow while $j = 1$ corresponds to axisymmetric flow. The subscript e denotes values of the parameters in the nonviscous flow. The remaining notation is standard in the literature. The presence in (1.1) of the integral calculated on the basis of the parameters of the nonviscous flow makes it possible, in approximate formulation, to account for the influence of the vorticity of the nonviscous flow on the interaction flow.

Solution of the problem of sub- and supersonic injection into a secondary super- and subsonic flow in a channel in an approximate formulation within the scope of the viscous—nonviscous interaction model reduces to the joint integration of the Euler equations, the boundary-layer equations, and conditions (1.1)-(1.3). With supersonic flow the Euler equations in the nonviscous region are of a hyperbolic type, where the longitudinal x coordinate functions as the regular coordinate, and the boundary-layer equations are of the parabolic type. Consequently, in order to calculate these flows we can formulate the Cauchy problem. However, the correctness of this problem with respect to the initial data depends on the structure of the flow in the viscous flow region. Since in the case of flows with reverse-circulation zones the Cauchy problem is incorrect for the boundary-layer equations, in the present study we have investigated only flows with a straightforward longitudinal velocity profile.

In the problems of viscous–nonviscous interaction, with the joining of solutions in corresponding subregions, there arises a mechanism for transmission of information upstream. We are dealing here with the fact that the extreme nature of the mathematical problem with an earlier unknown boundary (the surface of the effective displacement body) may impart "elliptical" properties to the solution. In this formulation this mechanism appears through the presence of a singular saddle-type point for system of conditions (1.1)-(1.3), while the singular solution of the system of given equations corresponds to the flow that is realizable from the physical standpoint. On the strength of the continuous relationship between the solutions of system (1.1)-(1.3) and the initial data, the unknown value of the static pressure at the inlet cross section corresponds to a singular solution.

For each integral curve (including a singular curve) in the regions of viscous and nonviscous flows it becomes necessary to integrate the equations of gasdynamics and of the boundary layer. The efficiency with which the standard schemes are utilized in problems of viscous interaction depends significantly on the means used to join the solutions. In contrast to the method of global iterations, requiring repeated sequential calculation of the field of flow in the corresponding regions, the utilization of system (1.1)-(1.3) allows us to construct a standard computational algorithm whose essence lies in the following. In the joint integration of the Euler equations, of the boundary layer, and of conditions (1.1)-(1.3), the derivative contained within these conditions is approximated with second-order accuracy by unilateral differences involving the use of pressure values at two points adjacent to the boundary and at a point that is part of the boundary. Since in the explicit finite-difference scheme the values of the gasdynamic parameters at points not lying at the boundary in the x_{n+1} section can be found from the values of the parameters at the x_n section and are independent of the conditions prevailing at the boundary surface, then on transition from x_n to x_{n+1} the parameters at the internal points of the nonviscous flow are found independently of the conditions of viscous–nonviscous interaction. The δ^* coordinate of the displacement body and the magnitude of the pressure P_e on that body in the x_{n+1} section are determined in first approximation from (1.1)-(1.3) for values of coefficients calculated from the parameters of the flow in the viscous region at the x_n section. The subsequent refinement of P_e and δ^* in the x_{n+1} section occurs in the iteration process and is associated with the nonlinearity of the boundary-layer equations. In addition to the indicated iterations, it is necessary to carry out global iterations in order to choose the pressures in the initial section, the appropriate choice for which will make it possible to pass through the singular point of conditions (1.1)-(1.3) and thus to construct a unique solution.

2. Let us examine the problem of calculating the flow in the near wake behind a body on discharge of a subsonic jet into a secondary unbounded supersonic flow. The parameters for the nonviscous flow were determined through integration of the gasdynamic equations in accordance with the finite-difference MacCormack scheme [11] on the basis of a developed complex of programs [12]. As in [10], the boundary-layer equations were integrated in accordance with the implicit finite-difference scheme in normalized Mises variables. We made use of algebraic and differential models of turbulence. We chose the Prandtl model as the algebraic model [13]

$$\nu_t = \varkappa|u_{\max} - u_{\min}|\delta \tag{2.1}$$

(the proportionality factor $\kappa = 0.27$ at the initial segment of the jet and it was equal to 0.22 in the main segment). For the differential model of turbulence we took the one-parameter model for turbulent viscosity, which has gained widespread acceptance [6, 7]:

$$\rho u \frac{\partial \nu_t}{\partial x} + \rho v \frac{\partial \nu_t}{\partial y} = \beta_0 f(\mathrm{M}) \rho \nu_t \left|\frac{\partial u}{\partial y}\right| + \xi \nu_t \left(u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y}\right) + \\ + \frac{1}{y^j} \frac{\partial}{\partial y}\left(y^j \frac{\rho \nu_t}{\mathrm{Pr}_{\nu_t}} \frac{\partial \nu_t}{\partial y}\right), \tag{2.2}$$

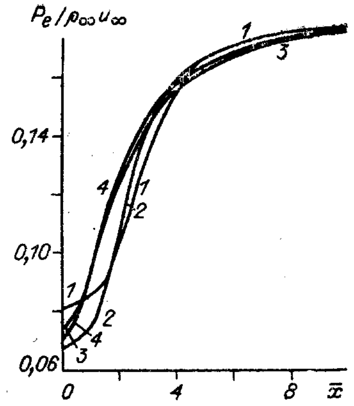

Fig. 2

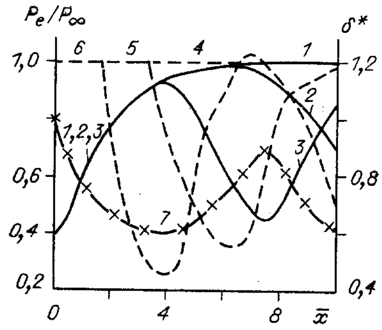

Fig. 3

where β_0, ξ, and Pr_{ν_t} are empirical constants ($\beta_0 = 0.2$, $\xi = 2/3$, $Pr_{\nu_t} = 0.5$), and for the function f(M) we use the approximation

$$f(M) = \begin{cases} 1 & \text{when } M \leqslant 1, \\ 1/M & \text{when } M > 1. \end{cases}$$

The distribution of static pressure along the axis of the plane symmetrical wake is shown in Fig. 2. Curves 1 and 2 correspond to the differential (2.2) and algebraic (2.1) models of turbulence, curve 3 corresponds to the flow with a constant turbulent viscosity $\bar{\nu}_t = \nu_t/(hu_\infty) = 0.01$, and curve 4 corresponds to calculation of the laminar wake at a characteristic Reynolds number of $Re = \rho_\infty u_\infty h/\mu_\infty = 100$ and an exponential relationship between viscosity and temperature, with an exponent $\omega = 0.5$. Here h is the width of the bottom outlet and u_∞ represents the velocity of the unperturbed outer flow. As we can see from a comparison of the curves, the turbulence models affect the magnitude of the pressure at the inlet section and weakly affect the distribution of pressure downstream beyond the blockage section. The results presented below have therefore been found by means of the algebraic model of turbulence, since it is the simplest.

The distinguishing factor of the problem of subsonic injection into a secondary supersonic flow in a channel or a tube is the formulation of the boundary conditions for nonpenetration into the theoretical region bounded by the surface AA. This corresponds to the problem of the discharge of a block of plane identical subsonic jets into a secondary uniform supersonic flow, when both the surface AA and the plane BB serve as the symmetry plane of the flow. The results obtained for this case can be interpreted as results from the calculation of the discharge of a subsonic jet into a secondary supersonic flow in a channel or tube, where friction at the walls is not taken into consideration. As demonstrated in [7], this assumption has little effect on pressure.

The distribution of the static pressure along the axis of the channel for various values of the relative height of the bottom recess in the case of plane flow is shown in Fig. 3 [curves 1-3: $\bar{h} = 0.1$, 0.334, and 0.5 ($\bar{h} = h/H$)]. The calculations have been carried out for the following determining parameters: the Mach number of the unperturbed outer flow is $M_\infty = 2$, the relative injection intensity for the subsonic jet is $\bar{q}_V = \rho_V u_V/\rho_\infty u_\infty = 0.05$, and the deceleration temperatures in the outer flow and in the jet are identical. The dashed lines in Fig. 3 show the pressure distributions at the channel wall (curves 4-6, respectively) and the boundary of the effective displacement body (7). From the pressure peaks on the characteristic flow lines we can judge the position and intensity of the compression shocks and the rarefaction waves formed in the nonviscous flow field. Comparison of the calculation results shows the extent to which the presence of a wall exerts significant influence on the wave structure of the flow. The larger $\bar{h}$, the more rapidly does the structure of the nonviscous flow affect the parameters of flow in the viscous region.

Given the earlier determining parameters, Fig. 4 shows the theoretical distribution of the static pressure in the viscous region in the channel for $\bar{h} = 0.5$ at a rather great distance downstream. We track the wavelike periodic behavior of pressure, with an amplitude attenuating downstream.

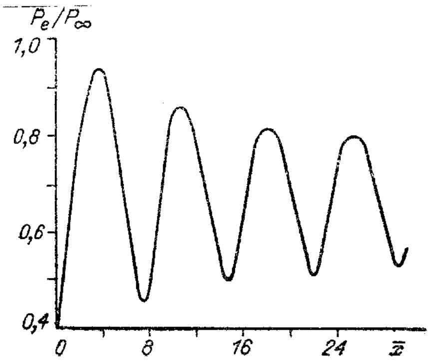

Fig. 4

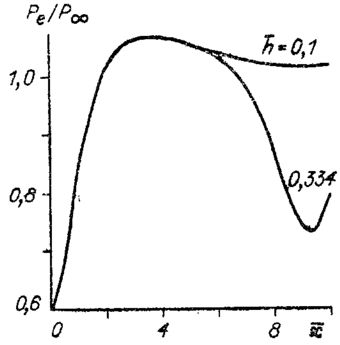

Fig. 5

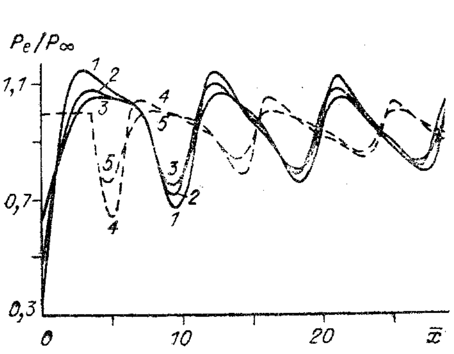

Fig. 6

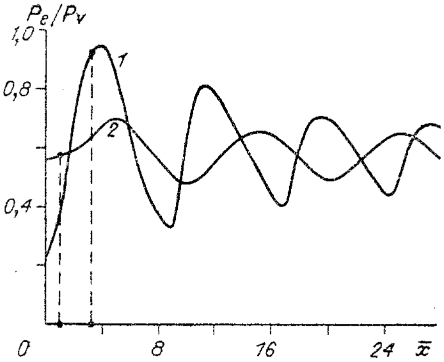

Fig. 7

The results of the numerical calculation of the static-pressure distribution in the viscous region along the axis of the tube for two values of the relative nozzle radius $\bar{h}$ = 0.1, 0.334 and $\bar{q}_V$ = 0.1 are shown in Fig. 5. The axisymmetric flow pattern is, on the whole, analogous to a plane pattern. However, there are certain differences. First of all, the pressure gradients in the case of axisymmetric flow are greater in magnitude than in the case of plane flow, and second, the maximum static pressure in the viscous flow region is greater than the static pressure in the nonviscous unperturbed flow.

Figure 6 shows the results of calculations which reflect the influence of injection intensity. The numerals 1-3 identify the static-pressure distribution curves along the jet axis with $\bar{q}$ = 0.05, 0.1, and 0.15, respectively, $\bar{h}$ = 0.334, and the previous conditions. The distribution of the static pressure along the wall of the tube when $\bar{q}$ = 0.05 and 0.15 is shown by curves 4 and 5. We see the degeneration of the wavelike character of the pressure distribution as $\bar{h}$ increases. This is associated with the increase in the bottom pressure $P_g = P_e(0)$ and, as a consequence, with the reduction in the intensity of the rarefaction waves at the edge of the nozzle, engendering the wavelike nature of the downstream changes in pressure.

Let us examine the problem of the interaction of a supersonic jet with a secondary subsonic flow in a channel. As in the previous problem, the parameters in the subsonic region are obtained by proceeding from the boundary-layer equations, and the injected supersonic jet is calculated by the finite-difference method with the aid of Euler equations. The pressure distribution in the subsonic flow is derived from the conditions of viscous–nonviscous interaction (1.1)-(1.3). The flow parameters in the underexpanded jet are determined through specification of the static pressure P_V, the deceleration temperature $T_V{}^0$, the Mach number M_V at the nozzle outlet, and in the external uniform subsonic flow through specification of the specific flow rate $q_\infty = \rho_\infty u_\infty$ and the deceleration temperature $T_\infty{}^0$. The unknown value of the static pressure P_∞ in the external subsonic flow is found during the process of solving the problem from the conditions of passage through the singular point of the system of equations of viscous–nonviscous interaction. Figure 7 shows the static-pressure distribution along the channel wall (as before, the effect of friction is neglected) for $M_V = 2$, $\bar{q}_\infty = \rho_\infty u_\infty/\rho_V u_V = 0.05$, $T_V{}^0 = T_\infty{}^0$. Curve 1 corresponds to calculation at $\bar{h}$ = 0.5, while curve 2 corresponds to calculation at $\bar{h} = 0.25$. The shape of the pressure curves is analogous to the corresponding distributions shown in Fig. 4 for the problems of interaction between a subsonic jet and a secondary supersonic flow. We observe the same wavelike distribution of pressure with an amplitude attenuating downstream. With an increase in the relative height of the nozzle, the oscillating amplitudes of the static pressure increases, while the period of oscillations diminishes, which is in agreement with the wave structure of the flow in the nonviscous underexpanded adjacent jet. The points in Fig. 7 identify the positions of the critical cross sections in the "throat" of the viscous region. Beyond these sections the viscous flow on the average becomes supersonic.

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